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LARGE-AMPLITUDE SOLITARY INTERNAL WAVES IN A TWO-LAYER FLUID

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UDC 532.539

The theoretical analysis of solitary waves at an interface separating two fluids of different densities is usually based on the Korteweg–de Vries equation [1-3], which has been derived not only on the standard assumption used in the theory of long waves, that the ratio of the fluid depth to the wavelength is small, but also with the additional assumption of relative smallness of the amplitude in comparison with the depth of the fluid. Consequently, the Korteweg–de Vries equation describes only small-amplitude solitary waves. Some experimental information on the range of validity of such modes may be found in [4]. A theoretical analysis of internal solitary waves without any constraints on their amplitude has been carried out in [5], where layers that move relative to one another are investigated in addition to layers that are at rest in the unperturbed state. Better experimental corroboration of the results of [5] has been obtained [4] for the velocity of wave propagation. The present article gives experimental data on the profiles of internal solitary waves, which are also in very good agreement with the model [5].

The waves were generated at an interface separating two layers of immiscible fluids of different densities, which were bounded below by a horizontal bottom and above by an impermeable horizontal cover plate. The principal notation and diagrams of the experimental arrangements are shown in Fig. 1a, b. Here H is the distance between the bottom and the cover plate, h_0 is the depth of the unperturbed lower layer, h is the depth of the perturbed lower layer, $\eta = h - h_0$ is the deviation of the interface from the equilibrium position, η_m is the amplitude, v is the velocity of propagation of the solitary waves, and ρ_0, $\rho < \rho_0$, u_0, and u are the densities and velocities of the lower and upper layers, respectively. A fixed xy rectangular coordinate system is used.

A rectangular duct with a working section of length 250 cm, width 18 cm, and height 6 cm (Fig. 1a) was used, as in [4], for the experimental creation of solitary waves in the case of fluids moving in the unperturbed state. The lower fluid could move with a velocity u_0 distributed uniformly along the vertical in the initial cross section, whereas only a slight circulatory motion took place in the upper layer in connection with friction at the interface. The working fluids were a dilute solution of salt (NaCl) in distilled water (ρ_0 = 1 g/cm³) and kerosene (ρ = 0.8 g/cm³). The waves were generated by a barrier in the form of a vertical plate set up at the exit from the duct and projecting above the bottom to a height b_1. Once a steady flow regime with depth h_0 of the lower fluid had been established, the barrier was raised smoothly to a height b_2 (for the generation of hummock-type waves) or was lowered (for the generation of crater-type waves) and was then brought back to its original position.

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 49-54, September-October, 1986. Original article submitted July 11, 1985.

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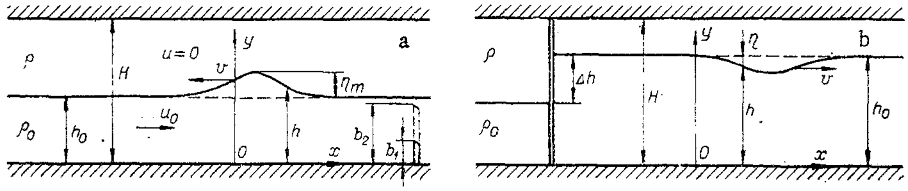

Fig. 1

The solitary waves in the case of layers initially at rest were investigated on a different experimental arrangement (Fig. 1b). It represented a duct of square cross section 6 × 6 cm and length 330 cm, which was divided into two parts by an impermeable partition. The depths of the lower fluid differed to the left and to the right of the partition. When the partition was quickly raised, solitary waves propagated in both directions away from it. If $h_0 \lesssim H/2$, a hummock-type wave moved in the positive x direction, and if $h_0 \gtrsim H/2$, (Fig. 1b), a crater-type wave moved in the same direction (the exact expression for the height h_0 at which the type of wave changes is given in [4]). The amplitude of the generated wave could be controlled by varying the speed of the partition and the initial level difference Δh at the partition.

The waves were measured by two electrical conductivity sensors similar to those used in [6], which were placed at distances of 75 and 100 cm from the generator. At this distance, the solitary wave had time to form completely, and undesirable ripple created by the motion of the wave generator died out. The propagation velocity v was measured according to the time for the crest (or trough) of the wave to traverse a distance of 25 cm. Repeated measurements under the same conditions showed that the random errors of estimation of the propagation velocity and amplitudes of the waves had coefficients of variation not greater than 1%.

The objective of the experiments was to compare calculated (according to the model of [5]) and experimental data on the profile and velocity of solitary waves. The analytical and experimental conditions were matched with respect to all the parameters of the problem except the viscosities of the fluids and the surface tension at the interface. Almost all the theoretical studies of solitary waves, including [5], have been carried out within the framework of the ideal-fluid model and without regard for surface tension. The kinematic viscosity coefficients in the experiments were 0.0108 cm^2/sec for water and 0.0162 cm^2/sec for kerosene, and the coefficient of surface tension was 34 dyn/cm.

Surface tension has scarcely any effect on long solitary waves [6], but it is useful in a number of respects. First, it enables us to obtain an abrupt change in the density and thus to match the experimental conditions ideally with the mathematical models. Second, it effectively suppresses the Kelvin–Helmholtz instability [7] (under the actual experimental conditons up to a velocity difference of about 19 cm/sec between the layers) and makes it possible to carry out investigations with layers in relative motion. Unwanted ripple created by the motion of the wave generator is also suppressed.

The viscosity of the fluids causes the amplitude, profile, and velocity of the waves to vary with distance from the source, i.e., they are nonstationary in general. However, the following quasistationary approach is applicable for liquids having a comparatively low viscosity (e.g., those used in the experiments) in the first approximation over a certain finite time interval. If the law governing the time variation of a particular parameter of the solitary wave is specified from the experimental results, its other parameters are predicted by the theory within the framework of the ideal-fluid model. Ordinarily, and this includes [5], the wave amplitude is taken as the free parameter. It was thus specified from the experiments. The mean values of the amplitude and the propagation velocity in the above-indicated interval between two sensors were used in comparison with the results of the calculations.

The results of an experimental confirmation of the model in [5] with respect to the wave velocity are given in [4]. Very good agreement with the theory is observed (within the limits of the previously indicated measurement error), whereas models based on an expansion in two small parameters give systematically overestimated values of v (as much as 10% too high). The experimental information on this problem has been augmented in the present study (in particular, waves propagating oppositely to the direction of the velocity u_0 are investigated); similarly good agreement is obtained with the model in [5].

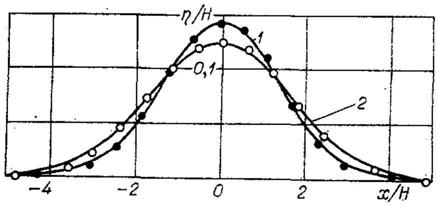

Fig. 2

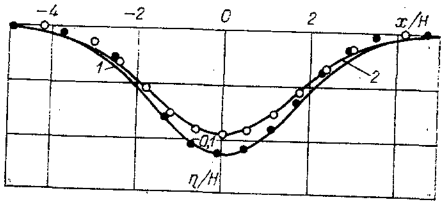

Fig. 3

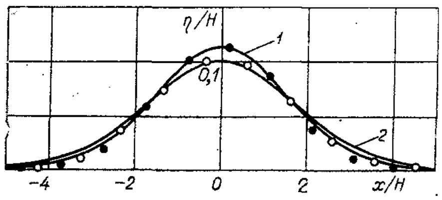

Fig. 4

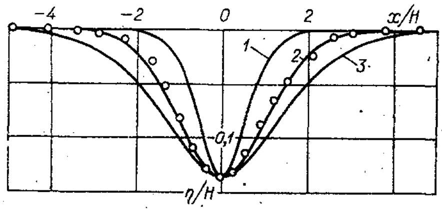

Fig. 5

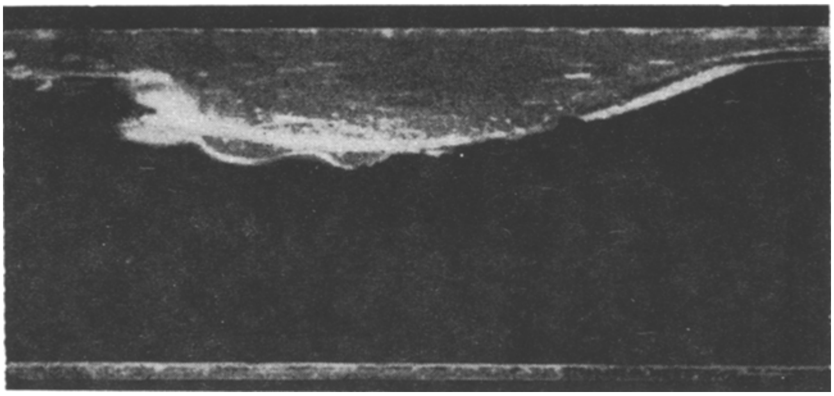

Fig. 6

Figures 2-4 show typical results of the experimental testing of the model in [5] for the solitary wave profiles. The following are obtained in [5] for hummock-type waves:

$$\frac{d\bar{y}}{d\bar{x}} = \sqrt{\frac{(\bar{y}-y_2)^2(y_4-\bar{y})(y_3-\bar{y})}{y_6(y_5-\bar{y})}}; \quad (1)$$

$$\bar{y} \simeq \frac{y_3 - y_4\theta^2 \mathrm{th}^2 k\bar{x}}{1-\theta^2 \mathrm{th}^2 k\bar{x}}, \quad (2)$$

where $\bar{y} = h/H$, $\bar{x} = x/H$, and y_i ($i = 2, \ldots, 6$), θ, and k are dimensionless constants determined by the values of the parameters of the problem H, h_0, η_m, ρ_0, ρ, and u_0 [5]. Analogous expressions apply to crater-type waves, differing only in the significance of the constants involved in them. We note that Eq. (2) is an approximate solution of (1) obtained in [5] by replacing the quantity $(y_5 - \bar{y})$ with a certain average constant. It was deemed important to check the errors incurred by such a replacement under real conditions. It was found that the discrepancy between the exact equation (1) and approximate solution (2) is practically indistinguishable in the scale of the graphs. The curves in Figs. 2-4 were plotted by the numerical solution of Eq. (1) on a computer.

Figure 2 gives the data for "hummock" waves with $u_0 = u = 0$. Curve 1 and the dark points are plotted for $y_0 = h_0/H = 0.315$; curve 2 and the light points are plotted for $y_0 = 0.35$. The experimentally determined wave velocity was found to be identical in both cases: $\bar{v} = v/\sqrt{gH} = 0.235$ (g is the acceleration of gravity). According to the calculations, $\bar{v}$ = 0.235 for curve 1, and $\bar{v}$ = 0.234 for curve 2.

Figure 3 illustrates the evolution of a crater-type solitary wave for $u_0 = u = 0$, $y_0 = 0.707$. The dark and light points and the corresponding calculated curves 1 and 2 refer to the same wave at distances from the partition $x/H = 12.5$ and 16.7. Viscous damping caused the amplitude to decay in the experiments. The calculated curves were varied accordingly, and the experimental values of the amplitude were used in the calculations. The velocity $\bar{v} = 0.233$ in the calculations and in the experiments between the two indicated cross sections.

Figure 4 shows the analogous evolution of a hummock-type solitary wave in the case of layers moving relative to one another for $u_0/\sqrt{gH} = 0.0835$, $u = 0$, $y_0 = 0.442$. The distance from the barrier $x/H = -13.3$ for curve 1 and the dark points, and $x/H = -17.5$ for curve 2 and the light points. In this interval, $\bar{v} = -0.190$ in the calculations and in the experiments.

It is evident from Figs. 2-4 that the model of [5] is in adequate agreement with the experimental data when the viscosity of the fluids is taken into account on the basis of the above-formulated quasistationary approach. The conditions of a "rigid cover" on the upper boundary of the layers are used in the model of [5] to arrive at reasonably simple final results. However, this model also gives sufficiently accurate results in the presence of a free surface if the depth of the upper layer is large in comparison with the wave amplitude; this fact is illustrated in Fig. 5, which shows the experimental data obtained for $H = 4.75$ cm, $u_0 = u = 0$, $y_0 = 0.737$. Curve 1 corresponds to the model of [2], curve 2 to the model of [5], and curve 3 to the model of [8]. We see that the model of [5] affords good agreement with the experimental data when the depth of the upper layer is only twice the value of the wave amplitude, whereas the model of [2] and [8] deviate strongly from the experimental. The wave velocity $\bar{v}$ in the given example is equal to 0.219 in the experiments, and its model values are 0.236 [2], 0.232 [5], and 0.225 [8].

The amplitude of the solitary waves at a density discontinuity cannot be too great (e.g., as a result of the Kelvin–Helmholtz instability). Even in the case of layers at rest in the unperturbed state, solitary-wave transmission can be accompanied by such a large velocity shear that surface tension is not adequate to suppress the indicated instability. This kind of situation is illustrated in Fig. 6, which shows an unstable internal solitary wave with a crater profile, which is traveling to the right. The instability that develops in the vicinity of the maximum velocity shear exerts a strong influence on the behavior of the wave as a whole. In particular, its leading edge can collapse. This wave dissipates rapidly.

In conclusion, we estimate the wave amplitude at which Kelvin–Helmholtz instability sets in. It can be shown on the basis of the laws of conservation of mass flow in each layer that the velocity shear between the layers in the cross section of maximum deviation of the interface from its equilibrium position is

$$\Delta u = \frac{aHv}{(H - h_m)\,h_m}, \quad v = \sqrt{\frac{\mu g h_m\,(H - h_m)}{(H - \mu h_m)}},$$

where $a = h_0 - h_m$, h_m is the depth of the lower fluid in this cross section, and the equation for v is taken from [4]. We see that the limiting wave amplitude for the onset of instability is

$$a_c = \Delta u_c \sqrt{h_m(H - h_m)(H - \mu h_m)/\mu g H^2}.$$

As mentioned, surface tension restricts the growth of the instability to a velocity difference $\Delta u_c = 19$ cm/sec. The maximum possible value of the wave amplitude in this case is $a_c = 1.58$ cm. Estimates of the parameters of the wave shown in Fig. 6 gives $a = 1.8$ cm and $\Delta u = 23$ cm/sec, which exceed the critical values.

The author is grateful to L. V. Ovsyannikov and V. I. Bukreev for furnishing the experimental data on their own initiative and for immeasurable assistance in the work, and also to O. A. Gavrilova for assistance in the numerical calculations on the computer.

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EVOLUTION OF THREE-DIMENSIONAL GRAVITATIONALLY WARPED WAVES DURING THE MOVEMENT OF A PRESSURE ZONE OF VARIABLE INTENSITY

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UDC 532.593:550.3

Three-dimensional, unestablished, gravitationally warped waves arising due to the motion of a harmonically time-varying pressure zone over a solid, thin plate floating on the surface of a homogeneous liquid of finite depth have been studied in the linear formulation. In the absence of a plate, three-dimensional waves are generated by the movement of a region of periodic perturbations, where established waves have been studied in [1, 2], and unestablished waves have been investigated in [3-5]. The evolution of three-dimensional, gravitationally warped waves formed during the motion of a constant load over a plate has been considered in [6].

1. We will consider a homogeneous, ideal, incompressible liquid of finite depth H covered by a thin, elastic plate. Beginning from time t = 0, a force of the following form acts on the surface of the plate:

$$p = p_0 f(x_1, y) \exp(i\sigma t),\ x_1 = x + vt,\ v = \text{const}. \tag{1.1}$$

We will investigate the evolution of excited wave motion assuming that the liquid is unperturbed up until the time when the force (1.1) acts and that the interface between the plate and the liquid (the flexure of the plate) ζ is horizontal.

Considering the motion of the liquid to be that of a potential and the velocity of the particles of the liquid and the elevation of the liquid–plate interface to be small, we will find in the coordinate system x_1, y, which is connected to a pressure zone moving with a velocity v, the velocity potential φ through the Laplace equation

$$\Delta\varphi = 0,\ -H < z < 0,\ -\infty < x < \infty,\ -\infty < y < \infty \tag{1.2}$$

with the following boundary and initial conditions

$$D_1\nabla^4\zeta + \varkappa_1 F\zeta + \zeta + (\varphi_t + v\varphi_x)\frac{1}{g} = -\frac{p}{\rho g}\quad (z = 0),$$
$$\varphi_z = 0\ (z = -H),\ \varphi(x, y, z, 0) = \zeta(x, y, 0) = 0, \tag{1.3}$$
$$D_1 = \frac{D}{\rho g},\quad \varkappa_1 = \frac{\rho_1 h}{\rho g},\quad D = \frac{Eh^3}{12(1-\mu^2)},\quad \nabla^4 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2\partial y^2} + \frac{\partial^4}{\partial y^4},$$
$$F = \frac{\partial^2}{\partial t^2} + 2v\frac{\partial^2}{\partial t\partial x} + v^2\frac{\partial^2}{\partial x^2},$$

Sevastopol. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 54-60, September-October, 1986. Original article submitted May 27, 1985.

 0021-8944/86/2705-0676$12.50